Posets of logics and the collapse arguments

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1. Introduction

2. Refining the collapse argument

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One can then rephrase the previous arguments.

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Claim: the notion of translation grants a sense of comparison between logics.

Mathematical and philosophical reasons for this setting.

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We can rephrase the collapse arguments as follows, even if we cannot decide anything about this ordering beforehand.

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Proposals for formal notions of translation: weak, LR and SR.

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Given two logics $\vdash_1, \vdash_2$, a *weak translation* from $\vdash_1$ into $\vdash_2$ is a map $\tau : \mathcal{L}(\vdash_1) \to \mathrm{Fm}(\vdash_2)$ that sends each *n*-ary connective $*$ to a *n*-ary term $\tau(*)$ in variables $x_1, \ldots, x_n$.

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Let $\tau : \vdash_1 \leqslant_{\mathrm{WT}} \vdash_2$. Given a $\mathcal{L}(\vdash_2)$-algebra $\mathcal{A}$, we define the τ-*translation* of $\mathcal{A}$ as the $\mathcal{L}(\vdash_1)$-algebra $\mathcal{A}^\tau$ of universe A in which each n-ary connective $*$ is interpreted as follows:

$$*^{\mathcal{A}^\tau}(a_1, \ldots, a_n) := \tau(*)^{\mathcal{A}}(a_1, \ldots, a_n),$$

for every $a_1, \ldots, a_n \in A$.

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4. Posets of logics: LR-translations

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Given a logic $\vdash$ and an $\mathcal{L}(\vdash)$-algebra $\mathcal{A}$, a subset $F \subseteq A$ is called a *deductive filter* of $\vdash$ on $\mathcal{A}$ if it is closed under valuations of inferences of $\vdash$: given $\Gamma \vdash \varphi$, for every $\mathcal{A}$-valuation v it holds that

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This gives rise to the so-called *strict Leibniz hierarchy*.

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Hence, the join of logics may not only not be admitted, but not even a proper logic in our sense above.

This means that, in principle, upward collapse is less likely to occur in $\mathrm{Log}_{\mathrm{SR}}$ than downward collapse.

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In $\mathrm{Log}_{\mathrm{SR}}$, a set of sufficient conditions for meet-irreducibility is known.

Moreover, very common logics are meet-irreducible: intermediate logics, Łukasiewicz, S4 and S5.

6. Future work: inside $\mathrm{Log}_{\mathrm{SR}}$

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2. Is it possible to develop a syntactical criterion for meet-irreducibility (again, check out the universal algebra side)?

3. Is is is possible to prove some factorization theorem (i.e. every logic is equi-translatable with an arbitrarily long meet of meet-irreducibles)?

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4. Study alternative notions of 'fundamental logic' (e.g. completely meet-irreducible logics).

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