Motivating logics of indicative conditionals

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Metis Seminar
Madrid, November 18, 2025

Outline

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We will denote the indicative conditional statement 'if φ then ψ' by $\varphi \to \psi$.

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Here, the intuitions may differ...

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Consider the algebra $\mathbf{DF}_3$ of universe $\{\mathbf{0}, \mathbf{1/2}, \mathbf{1}\}$ and operations:

	$\neg$
$\mathbf{0}$	$\mathbf{1}$
$\mathbf{1/2}$	$\mathbf{1/2}$
$\mathbf{1}$	$\mathbf{0}$

$\wedge_K$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{1/2}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1/2}$
$\mathbf{1}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$

$\vee_K$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
$\mathbf{1/2}$	$\mathbf{1/2}$	$\mathbf{1/2}$	$\mathbf{1}$
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$\to_{DF}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
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We define *Reichenbach-De Finetti logic* as $\mathrm{DF} := \mathrm{Log}(\mathbf{DF_3}, \{\mathbf{1/2}, \mathbf{1}\})$.

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(ii) the latter sanctions classically false implications:

2.17 *If the new manager is successful, he will be promoted, or if he is not successful, he will be transferred. Therefore, the new manager will either be promoted or transferred.*

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$(S \to P) \vee (\neg S \to T) = (S \vee \neg S) \to (P \vee T) = (P \vee T).$

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$\rightarrow_{\mathsf{OL}}$	$\mathbf{0}$	$^1\!/_2$	$\mathbf{1}$
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We define *Cooper's ordinary language logic* as $\mathrm{OL} := \mathrm{Log}(\mathbf{O}_3, \{\mathbf{1/2}, \mathbf{1}\})$.

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$\mathbf{F}_3$: $\neg, \wedge_\mathsf{K}, \vee_\mathsf{K}$ with

$\to_\mathsf{F}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
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(b) **Cantwell's logic of conditional negation:** interpret the previously considered three-valued negation as a 'conditional negation'. Then, $\neg\varphi$ is interpreted as 'φ is false if it has (classical) truth-value'. Let $\mathrm{CN} := \mathrm{Log}(\mathbf{CN}_3, \{\mathbf{1/2}, \mathbf{1}\})$, where

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$\to_{\mathsf{OL}}$	$\mathbf{0}$	$\mathbf{1/2}$	$\mathbf{1}$
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We distinguish between **intrinsic** (theoretical, intuitive, pre-formal) and **extrinsic** (technical, pragmatical, formal) motivations.

Normally, extrinsic motivations point at properties that a logic should verify while intrinsic ones show how the three-valued setting extends the two-valued one.

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$\varphi \wedge_{\mathsf{K}} \psi = \mathbf{1}$ iff $\varphi = \mathbf{1}$ and $\psi = \mathbf{1}$,

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Cooper conjunction:

$\varphi \wedge_{\mathsf{OL}} \psi = \mathbf{1}$ iff $(\varphi = \mathbf{1}$ and $\psi \neq \mathbf{0})$ or $(\varphi \neq \mathbf{0}$ and $\psi = \mathbf{1})$,

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Farrell conditional:

$\varphi \rightarrow_{\mathsf{F}} \psi = \mathbf{1}$ iff $\varphi = \mathbf{1}$ and $\psi = \mathbf{1}$,

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So, at least extrinsically, we would lose by restricting to the only-truth scenarios.

Alternative: even in the non-falsity cases, why not setting $\mathbf{1/2} \to \mathbf{1/2} = \mathbf{1}$?

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If I have enough time and money, I will go to the cinema. Hence, if I have enough time I will go to the cinema or if I have enough money I will go to the cinema.

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Either I don't travel or, if travel, then I spend a lot. Hence, either I don't travel or I spend a lot. DF and F falsify this!

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$$\mathbf{0} \mapsto (0,1), \quad \mathbf{1} \mapsto (1,0), \quad \mathbf{1/2} := \top \mapsto (1,1),$$

where we understand each component as a switch: the first one is a *truth-switch* and the second one a *falsity-switch*.

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We can distinguish two approaches by the behavior of negation:

- $\perp$ behaves like a **gap**: $\neg\top = \top$ and $\neg\perp = \perp$.
- $\perp$ behaves like a **falsity**: $-\top = \perp$ and $-\perp = \top$.

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Thus, $\top$ and $\bot$ are better seen as encoding semantic *gluts* and *gaps*, respectively. Note that we have refined the original intuition on 'semantic gaps'!

Why do we also preserve $\top$? These logics are *logics of information*.

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A more elegant way of introducing 4-valued logics is by introducing a propositional predicate P and considering:

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Hence, $\top$ and $\bot$ correspond to *vacuously true* and *vacuously false*, respectively.

Are the resulting logics of truth or of non-falsity?

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- We have analyzed how the conectives four-valued logics of indicative conditionals have been extrinsically motivated.

- We have studied intrinsic motivations for the new truth-value, at least with respect to negation and regarding the designated elements.

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